Solution by J. SCHEFFER, A. M., Hagerstown, Md., and the PROPOSER.

Let $x=2a\tan^2\theta$, $y=a\tan\phi$, $m=\frac{1}{2}(2n-1)$.

$$\therefore A=\frac{2}{(2a)^{n-\frac{1}{2}}}\int_0^{\frac{1}{2}\pi}\cos^{2(n-1)}\theta\, d\,\theta=\frac{1}{2(2a)^m}\int_0^{\frac{1}{2}\pi}\cos^{2m-1}\theta\, d\,\theta=\frac{\sqrt{(\pi)}\Gamma(m)}{(2a)^m\Gamma\left(\frac{2m+1}{2}\right)}.$$

$$B=\frac{1}{a^{n-\frac{1}{2}}}\int_0^{\frac{1}{2}\pi}\tan^{n-\frac{1}{2}}\phi\cos^{2(n-1)}\phi\, d\phi=\frac{1}{a^m}\int_0^{\frac{1}{2}\pi}\sin^m\phi\cos^{m-1}\phi\, d\,\phi$$

$$=\frac{\Gamma\left(\frac{m+1}{2}\right)\,\Gamma\left(\frac{m}{2}\right)}{2a^m\,\Gamma\left(\frac{2\,m+1}{2}\right)}.$$

But $\Gamma\left(\frac{m+1}{2}\right)\,\Gamma\left(\frac{m}{2}\right)=\frac{\sqrt{(\pi)}\Gamma(m)}{2^{m-1}}$.

$$\therefore B=\frac{\sqrt{(\pi)}\Gamma(m)}{(2a)^m\Gamma\left(\frac{2\,m+1}{2}\right)}=A.\quad \text{Hence } A/B=1.$$

Exhaustive solutions, though differing in results from the one given here, were received from M. V. Spunar, Francis Rust, and T. G. Wodo.

258. Proposed by A. H. HOLMES, Brunswick, Maine.

Evaluate $\displaystyle\int_0^{\frac{1}{2}a}\frac{dx}{\sqrt{[2ax-x^2\sqrt{(a^2-x^2)}]}}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $x=x\sin\theta$. Then

$$I=\int_0^{\frac{1}{2}a}\frac{dx}{\sqrt{[2ax-x^2\sqrt{(a^2-x^2)}]}}=\int_0^{\frac{1}{6}\pi}\frac{\cos\theta\, d\,\theta}{\sqrt{[2\sin\theta-a\sin^2\theta\cos\theta]}}.$$

$$\therefore I=\frac{1}{\sqrt{2}}\int_0^{\frac{1}{6}\pi}\Big[1+\frac{a}{4}\sin\theta\cos\theta+\frac{3a^2}{32}\sin^2\theta\cos^2\theta+\frac{5a^3}{128}\sin^3\theta\cos^3\theta$$

$$+\frac{35a^4}{2048}\sin^4\theta\cos^4\theta+\frac{64a^5}{8192}\sin^5\theta\cos^5\theta+\frac{231a^6}{65536}\sin^6\theta\cos^6\theta+\ldots\Big]\frac{\cos\theta\, d\,\theta}{\sqrt{(\sin\theta)}}$$

$$=1+\frac{31}{15}\cdot\frac{a^2}{2^9}+\frac{85015}{1989}\cdot\frac{a^4}{2^{19}}+\frac{2350494}{38675}\cdot\frac{a^6}{2^{28}}+\ldots.+\frac{1}{\sqrt{2}}\int_0^{\frac{1}{6}\pi}\Big[\frac{a}{4}\sin\theta\cos\theta$$

$$+\frac{5a^3}{128}\sin^3\theta\cos^3\theta+\frac{63a^5}{8192}\sin^5\theta\cos^5\theta+\ldots\Bigg]\frac{\cos\theta\, d\theta}{\sqrt{(\sin\theta)}}.$$

$$\frac{1}{\sqrt{(\sin\theta)}}=(1-\cos^2\theta)^{-\frac{1}{4}}.$$

$$\therefore I=1+\frac{31}{15}\cdot\frac{a^2}{2^9}+\frac{85015}{1989}\cdot\frac{a^4}{2^{19}}+\frac{2350494}{38675}\cdot\frac{a^6}{2^{28}}+\ldots$$

$$+\frac{1}{\sqrt{2}}\Bigg[\frac{a}{35.\,2^5}\left(\frac{341}{3}-\frac{20611\sqrt{3}}{2^9}+\ldots\right)+\frac{a^3}{21.\,2^8}\left(\frac{41}{3}-\frac{2616\sqrt{3}}{2^{10}}+\ldots\right)$$

$$+\frac{a^5}{11.\,2^{10}}\left(1-\frac{8181\sqrt{3}}{2^{14}}+\ldots\right)\Bigg]+\ldots$$

a cannot be greater than $\frac{8}{9}\sqrt{3}$.

This solution, to be complete, should have investigated the matter of convergency and, since the function vanishes at the lower limit, also the condition of determinateness.

The proposer of 259 (256) suggests that the equation be changed to $(1+y+2axy)dx+x(1+x)dy=0$.

MECHANICS.

211. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A smooth elliptic wire, axis vertical, has a small ring sliding on it, connected by elastic strings with each focus. Either string is just unstretched when the ring is nearest the corresponding focus. The modulus of elasticity is W/n, where W oz. is the weight of the ring. Find the distance of the ring from the upper focus in the different positions of equilibrium, and in each case discuss the nature of the equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let F be the upper focus of the elliptic wire, semi-major axis a, eccentricity e. Let r, $2r-a$ be the lengths of the strings from the upper and lower foci to the ring at the point P on the curve; G the intersection of the normal from P with the axis major; θ the angles the strings make with the tangent at P; ϕ the angle the tangent at P makes with the major axis; T the tension of string r; Q the tension of string $2a-r$; m the weight of the ring and strings that cause a downward force due to gravity; $a(1-e)$=unstretched length of each string.

Then $PF=r$, $GF=er$, $\sin GPF=\cos\theta$, $\sin PGF=\cos\phi$, $\sin PGF:\sin GPF=r:er$. $\therefore \cos\phi/\cos\theta=1/e$.

Also $r=a(1-e)(1+Tn/W)$, $2a-r=a(1-e)(1+Qn/W)$.